Addendum 1

Speakers' knowledge of phonological universals: Evidence from nasal clusters

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In this analysis we assume that for any underlying form /uf/ there is a set of surface forms faithful to /uf/. There may be multiple such forms because /uf/ may be underspecified to some degree; e.g., prosodic structure may not be present in /uf/. We assume a surface form [sf] to be fully specified in the sense that it uniquely determines a phonetic form |φf| which is the fully faithful phonetic interpretation of [sf]; |φf| in turn determines a unique auditory form {af} which results from the fully faithful execution of |φf|. Conversely, we assume that if $[sf_1] \neq [sf_2]$ then the phonetic forms that are fully faithful to them, $|sf_1|$ and $|sf_2|$, are not equal, and analogously, if $|sf_1| \neq |sf_2|$ then the acoustic forms fully faithful to them, $\{af_1\}$ and $\{af_2\}$, are not equal. Assumption 1 spells these out in the notation of the text.

Assumption 1. Faithful forms: f

For any underlying form /uf/ there is a set f(/uf/) of surface forms faithful to /uf/: for each [sf] ∈ f(/uf/), the pair (/uf/, [sf]) satisfies the faithfulness constraint $F_{/uf/,[sf]}$.

For any surface form [sf] there is a unique phonetic form f([sf]) that is faithful to [sf]: ([sf],f([sf])) satisfies $F_{[sf],|\varphi f|}$. This function f is one-to-one. f([sf]) is also said to be *the phonetic form corresponding to* [sf].

For any phonetic form |φf| there is a unique auditory form f(|φf|) that is faithful to |φf|: (|φf|, f(|φf|)) satisfies $F_{|\varphi f|,\{af\}}$. This function f is one-to-one. f(|φf|) is also said to be *the acoustic form corresponding to* |φf|.

Def. Grammatical. With respect to a given grammar $\mathcal{G}$:

A surface form [sf] is *grammatical* iff there is an underlying form /uf/ such that the pair (/uf/, [sf]) is optimal: for any other surface form [sf′], the grammar assigns lower harmony to (/uf/, [sf′]) than to (/uf/, [sf]): (/uf/, [sf′]) $\prec_{\mathcal{G}}$ (/uf/, [sf]).

A phonetic form is grammatical iff it corresponds to a grammatical surface form.

An acoustic form is grammatical iff it corresponds to a grammatical phonetic form.

Suppose we are given a phonetic form $|\varphi f_u|$ and its corresponding acoustic form $\{af_u\} = f(|\varphi f_u|)$. We are interested in cases where $|\varphi f_u|$ is ungrammatical for a hearer, that is, where there is no surface form $[sf_g]$ that is grammatical for the hearer such that $|\varphi f_u|$ is the faithful phonetic interpretation of $[sf_g]$. In this case, we assume that among the grammatical surface forms, one, call it $[sf_g]$, is most faithful to $|\varphi f_u|$. That is, given any other grammatical surface form $[sf_g']$, the pair ($[sf_g]$, $|\varphi f_u|$) better-satisfies faithfulness $F_{[sf],|\varphi f|}$ than does the pair ($[sf_g']$, $|\varphi f_u|$); we write this:

$$F_{[sf],|\varphi f|}([sf_g], |\varphi f_u|) \succ F_{[sf],|\varphi f|}([sf_g'], |\varphi f_u|).$$

We assume that the phonetic interpretation of this surface form, $|\varphi f_g| = f([sf_g])$, is the grammatical phonetic form that is most faithful to the ungrammatical acoustic form $\{af_u\}$ corresponding to $|\varphi f_u|$.

Assumption 2. Given any phonetic form |φf$_0$| and its corresponding acoustic form {af$_0$} = f(|φf$_0$|):

Among grammatical surface forms, one, [sf$_g$], is most faithful to |φf$_0$|; i.e., [sf$_g$] best satisfies $F_{[sf],|\varphi f|}$([sf],|φf$_0$|) among grammatical [sf].

The phonetic form corresponding to [sf$_g$], |φf$_g$| = f([sf$_g$]), is the grammatical phonetic form that is most faithful to the given acoustic form {af$_0$}: |φf$_g$| best satisfies $F_{|\varphi f|,\{af\}}$(|φf|, {af$_0$}) among grammatical |φf|.

We assume given an OT grammar $\mathcal{G}$ for the mapping between underlying and surface forms. The Harmonic Ordering of Forms determined by $\mathcal{G}$ (Prince & Smolensky 1993/2004: Ch. 5) will be written as follows: when $\mathcal{G}$ assigns higher Harmony to (/x/, [x]) than to (/y/, [y]) we write (/x/, [x]) $\succ_{\mathcal{G}}$ (/y/, [y]) or equivalently H_1(/x/, [x]) $\succ$ H_1(/y/, [y]) since this is Harmony in the first component. Harmony in the second component, linking surface and phonetic forms, is determined solely by the faithfulness constraint $F_{[sf],|\varphi f|}$, so H_2([x], |x|) $\succ$ H_2([y], |y|) means that ([x], |x|) better satisfies faithfulness than does ([y], |y|): $F_{[sf],|\varphi f|}$([y], |y|) $\succ$ $F_{[sf],|\varphi f|}$([y], |y|). In the same way, Harmony in the third component, linking phonetic and acoustic forms, is determined by $F_{|\varphi f|,\{af\}}$; H_3(|x|, {x}) $\succ$ H_3(|y|, {y}) means that $F_{|\varphi f|,\{af\}}$(|x|, {x}) $\succ$ $F_{|\varphi f|,\{af\}}$(|y|, {y}). All the analogous definitions apply to $\prec$ and to $\approx$ (equal Harmony), as well as to $\succcurlyeq$ and $\preccurlyeq$.

Optimality in component *n* is the obvious generalization of the component-1 notion of optimality in OT.

Def. Optimal in a component *n*

(x_n, x_{n+1}) is optimal in component *n* ("*n*-opt") iff for all level-*n*+1 forms $x_{n+1}' \neq x_{n+1}$,
$H_n(x_n, x_{n+1}) \succ H_n(x_n, x_{n+1}')$

Def. Harmonic ordering w.r.t. component *n*

Given two representations X = (/x/, [x], |x|, {x}) ≡ (x_1, x_2, x_3, x_4) and Y = (y_1, y_2, y_3, y_4):

X $\succ_n$ Y iff either

(i) (x_n, x_{n+1}) is optimal and (y_n, y_{n+1}) is not, or

(ii) (x_n, x_{n+1}) and (y_n, y_{n+1}) are both sub-optimal and $H_n(x_n, x_{n+1}) \succ H_n(y_n, y_{n+1})$

X $\approx_n$ Y iff either

(i) (x_n, x_{n+1}) and (y_n, y_{n+1}) are both optimal, or

(ii) (x_n, x_{n+1}) and (y_n, y_{n+1}) are both sub-optimal and $H_n(x_n, x_{n+1}) \approx H_n(y_n, y_{n+1})$

X $\succcurlyeq_n$ Y iff either X $\succ_n$ Y or X $\approx_n$ Y

To combine harmonic ordering across all components, we assume that no component has priority over any others. Thus for a full (four-level) representation X to be preferred to Y, written X $\succ$ Y, there must be some component on which X is preferred and no component on which Y is preferred. The ordering is partial in that for many pairs X, Y that are not harmonically equivalent, neither X $\succ$ Y nor Y $\succ$ X. Our ordering is defined for use in perception: it compares X and Y only if they have the same acoustic form {af}, and are therefore both potential full perceptual representations for {af}.

Def. Harmonic ordering of full representations: $\succ$

Given two representations X = (/x/, [x], |x|, {x}), Y = (/y/, [y], |y|, {y}):

X $\succ$ Y iff {y} = {x} and either

(a) i. $\forall k$ [Y is k-opt $\Rightarrow$ X is k-opt] &

ii. $\exists n$ [X is n-opt and Y is not n-opt]

or

(b) i. $\forall k$ [Y is k-opt $\Leftrightarrow$ X is k-opt] &

ii. $\forall k$ [X is not k-opt $\Rightarrow$ X $\succcurlyeq_k$ Y] &

iii. $\exists n$ [X $\succ_n$ Y]

In case (a), optimality makes the decision: X is optimal in some component in which Y is not (a.ii), and there is no component in which Y is optimal but X is not (a.i). In case (b), X and Y tie with respect to optimality, in that they are optimal in exactly the same components (b.i): then relative Harmony among sub-optimal representations decides. Now X is preferred to Y only if there is some component in which X is sub-optimal but higher-Harmony than Y (b.iii), and there is no component in which Y is sub-optimal but higher-Harmony than X (b.ii).

Henceforth references such as (b.ii) will always refer to the correspondingly labeled condition in the definition of $\succ$.

Lemma 1. Suppose X is optimal in every component. Then

(α) there is no Y such that Y $\succ$ X, and

(β) for every Z such that {z} = {x}, X $\succ$ Z unless Z is also optimal in every component.

Proof of Lemma 1. Suppose there were a Y s.t. Y $\succ$ X. Then either (a.ii) $\exists n$ [Y is n-opt and X is not n-opt], which is impossible because X is k-opt $\forall k$, or (b.iii) $\exists n$ [Y $\succ_n$ X], which is also impossible because X is k-opt $\forall k$. This establishes (α). Now suppose given Z such that {z} = {x}. Suppose Z is not optimal in every component: $\exists n$ [Z_n is sub-opt]. Then (a) of the definition of X $\succ$ Z is satisfied, establishing (β). QED

Lemma 2. Suppose X is suboptimal in component k and optimal in the other components. Let Y be a candidate with {y} = {x}. Then

1. X $\succ$ Y iff either

a. Y is [i] suboptimal in some non-k component and [ii] not k-optimal or

b. Y is [i] optimal in all non-k components and [ii–iii] X $\succ_k$ Y,

2. Y $\succ$ X iff [Y is optimal in all non-k components and Y $\succ_k$ X]

3. neither X $\succ$ Y nor Y $\succ$ X if [Y is optimal in component k and suboptimal in some non-k component].

Proof of Lemma 2.

Suppose X is suboptimal in component k and optimal in the other components, and Y is a candidate with {y} = {x}.

Part 1. Under these conditions, conditions 1.a and 1.b of Lemma 2 are equivalent to (a) and (b), respectively, of the definition of X $\succ$ Y.

Part 2. Exchanging X and Y in 1, we see that 1.a cannot be met and condition 1.b becomes 2 of Lemma 2.

Part 3. Suppose now that Y is optimal in component *k* and suboptimal in some non-*k* component. Then in the definition of X ≻ Y, condition (i) cannot hold in either (a) or (b), and the same is true for the definition of Y ≻ X, because in both (a) and (b), (i) requires that the set of components in which one of the candidates X or Y is optimal is a (not necessarily strict) subset of the components in which the other candidate is optimal. This establishes result 3. QED

Now, as in (4) in the text, we define a possible percept as follows.

Def. Perceptual principle

Let X = (/x/, [x], |x|, {x}) be a globally faithful representation and consider the auditory form {x}. A representation Y is a *possible percept for* {x} if and only if

the acoustic form of Y is {x}, and

there is no Z with acoustic form {x} such that Z ≻ Y.

The main result, (5) in the text, follows.

Proposition:

Let X = (/x/, [x], |x|, {x}) be a globally *faithful* representation. For the auditory input {x}, there are two possibilities.

a. If [x] is grammatical then the only possible percept type is

A′ = (/x′/, [x], |x|, {x})

where /x′/ is any underlying form for which (/x′/, [x]) is *optimal*.

b. If [x] is not grammatical, there are three possible percept types:

A′ = (/x′/, [x], |x|, {x})

C′ = (/y′/, [y], |x|, {x})

D′ = (/y′/, [y], |y|, {x})

where

[y] is the grammatical surface form most faithful to |x|,

|y| = *f*([y]) is the phonetic form faithful to [y],

/y′/ is any underlying form for which (/y′/, [y]) is *optimal*, and

/x′/ ∈ *f*([x]) is any underlying form *faithful* to [x].

Proof. Part a: grammatical case. That A′ (as given in the Proposition) is a possible percept follows immediately from (α) of Lemma 1 since A′ is optimal in every component: (/x′/, [x]) is optimal by assumption, and in the other two components, A′ is faithful hence optimal.

Now consider any possible percept U = (/u/, [u], |u|, {u}); we must show it is of type A′. As a possible percept of {x}, U must have {u} = {x} and we must not have A ≻ U where A is a candidate of type A′. Now according to Lemma 1 (β), since A is optimal in every component, we would have A ≻ U unless U is also globally optimal. To be optimal in the third component, since {u} = {x}, |u| must be faithful to {x}, so, by Assumption 1, we must have |u| = |x|, as |x| is the unique phonetic form faithful to {x}: {x} = *f*(|x|). By identical reasoning, to be optimal in the second component, [u] must be faithful to |u| = |x| so we must have [u] = [x]. To be optimal in the first component, /u/ must be optimal when paired with [x]; this means U is a candidate of type A′.

Part b: ungrammatical case.
U = (/u/, [u], |u|, {u}) is a possible percept only if {u} = {x}. There are three cases to consider. In case 1, |u| = |x| and [u] = [x]; in case 2, |u| = |x| and [u] ≠ [x]; in case 3, |u| ≠ |x|. We show that in case 1, U is a possible percept iff it is of type A′; in case 2, type C′; in case 3, type D′. Note that the globally faithful candidate X is optimal in components 2 and 3 (as it is faithful there) but cannot be optimal in component 1 because [x] is ungrammatical: there is no /x′/ such that (/x′/, [x]) is optimal. X thus meets the conditions of Lemma 2, with $k = 1$. While not optimal in component 1, X is however faithful in component 1, by definition.

Case 1. Here, U = (/u/, [x], |x|, {x}). U is a possible percept iff there is no Z with acoustic form {x} such that Z ⋗ U. Like X, U is optimal in components 2 and 3 but cannot be optimal in component 1. So U satisfies the conditions of Lemma 2, with $k = 1$. By Part 2 of Lemma 2, for a candidate Z with acoustic form {x}, Z ⋗ U iff Z is optimal in components 2 and 3 and Z $\succ_1$ U. This is true iff Z = (/z/, [x], |x|, {x}) and (/z/, [x]) $\succ_1$ (/u/, [x]). In this Harmony comparison, the two pairs tie on MARKEDNESS since they have the same surface form [x]; (/z/, [x]) $\succ_1$ (/u/, [x]) will hold iff (/z/, [x]) satisfies FAITHFULNESS better than does (/u/, [x]). For no such Z to exist, it is necessary and sufficient that /u/ is fully faithful to [x]: if /u/ is fully faithful to [x], no /z/ can be more faithful, and if /u/ is not faithful to [x], then any /z/ that is faithful to [x] will satisfy (/z/, [x]) $\succ_1$ (/u/, [x]). Thus, under the conditions of Case 1, U is a possible percept iff it is of the form A′ as defined in Part 2 of the Proposition.

Case 2. Now, U = (/u/, [u], |x|, {x}) with [u] ≠ [x]. U is optimal (faithful) in component 3 but is sub-optimal in component 2 since (by Assumption 1) [x] is the unique surface form faithful to |x|. Since X meets the conditions of Lemma 2 with $k = 1$, and U is 2-sub-optimal, by part 1 of Lemma 2, we will have X ⋗ U iff U is not 1-optimal (this is 1.a; 1.b cannot be met by U). So to be a possible percept it is necessary that (/u/, [u]) be optimal, i.e., [u] must be a grammatical surface form and /u/ a possible underlying form for [u]. This means that U satisfies Lemma 2 with $k = 2$. Now U is a possible percept for {x} iff there is no candidate Z with acoustic form {x} such that Z ⋗ U. By part 2 of Lemma 2, such a Z ⋗ U iff Z is optimal in components 1 and 3 and Z $\succ_2$ U, which is true iff Z $\succ_2$ U and Z = (/z/, [z], |x|, {x}) with (/z/, [z]) optimal. This in turn is true iff [z] is grammatical and ([z], |x|) is more faithful than ([u], |u|). If we choose [u] to be the grammatical surface form [y] that is most faithful to |x| (unique by Assumption 2) then no such [z] exists, otherwise, choosing [z] = [y] entails Z ⋗ U. Thus, under the conditions of Case 2, U is a possible percept for {x} iff [u] = [y] and /u/ is an underlying form /y/ for which (/y/, [y]) is optimal, that is, iff U is a candidate of type C′.

Case 3. Now U = (/u/, [u], |u|, {x}) where |u| ≠ |x|. U is suboptimal in component 3 since (by Assumption 1) |x| is the only phonetic form that is faithful to {x}. Since X satisfies Lemma 2 with $k = 1$ and U is 3-sub-optimal, by part 1 of Lemma 2 we will have X ⋗ U iff U is not 1-optimal (this is 1.a; 1.b cannot be satisfied by such a U). So, as in case 2, for U to be a possible percept of {x} we must have (/u/, [u]) optimal, so [u] must be a grammatical surface form. Let W be a candidate of type C′. W satisfies Lemma 2 with $k = 2$ so again by part 1 of Lemma 2, W $\succ$ U iff U is not 2-optimal; U must be 2-optimal to be a possible percept, which means ([u], |u|) must be faithful. This means U satisfies Lemma 2 with $k = 3$. So by part 2 of that lemma, there exists a candidate Z with acoustic form {x} such that Z $\succ$ U iff there exists Z optimal in components 1 and 2 and Z $\succ_3$ U, i.e., a Z = (/z/, [z], |z|, {x}) such that [z] is a grammatical surface form with underlying form /z/, |z| = f([z]), and (|z|, {x}) is more faithful than (|u|, {x}). Such a Z does not exist if |u| is the (unique) grammatical phonetic form |y| most faithful to {x}, otherwise, such a Z does exist, namely, where |z| = |y|. By Assumption 2, this |y| = f([y]), where [y] is as defined in the Proposition. Thus, under the conditions of Case 3, U is a possible percept for {x} iff U is of type D′.

QED

Note: The second part of Assumption 2 can be dropped if D′ in the Proposition is set to (/z′/, [z], |z|, {x}), where |z| is the grammatical phonetic form most faithful to {x}, [z] is the surface form faithful to |z|, which is grammatical by definition of "grammatical phonetic form", and /z′/ is an underlying form for which (/z′/, [z]) is optimal, which exists since [z] is grammatical. Now the [z] of D′ may not be the same as the [y] of B′.

This file is http://www.cogsci.jhu.edu/faculty/smolensky/BerentEtAlPhonology09Addendum1.pdf.

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